Using Dataflow Optimization Techniques with a Monadic Intermediate Language

Justin Bailey
jgbailey@codeslower.com

Portland State University

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Introduction

- Compiling Functional Programming Languages
- Dataflow Analysis
- Three-Address Code

Dataflow Analysis for Functional Languages?

- Can we apply functional-language specific optimizations?
- Can we implement traditional dataflow-based optimizations?

Monadic Programming to the Rescue!

- Simple control-flow
- Separate side-effecting computation from "pure" values
- Higher-order functions

MIL: A Monadic Intermediate Language

- Monadic: Haskell's **do** notation.
- Monadic: Segregated side-effects.
- Dataflow: Basic blocks.
- Dataflow: Block scope.
- Dataflow: Based on three-address code.

Contributions

- Applied the *dataflow algorithm* to a functional language.
- Implemented *uncurrying*, using the dataflow algorithm.
- Thorough exposition of the HOOPL library.

Goals of MIL

- Simplicity
- Allocation & Other Side-Effects
- Higher-Order Functions

Simplicity

$$\frac{(b * c + d)}{2}$$

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$$\frac{(b * c + d)}{2}$$

- Three-Address Code

```
1  t1 := b * c;
2  t2 := t1 + d;
3  t3 := t2 / 2;
```

Simplicity

$$\frac{(b * c + d)}{2}$$

- MIL

```
1  t1 <- mul*(b, c)
2  t2 <- add*(t1, d)
3  t3 <- div*(t2, 2)
```

Side-Effects

Side-Effects

- Closures

```
t1 <- k{x}
```

Side-Effects

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```
t1 <- k{x}
```

Side-Effects

- Closures

```
t1 <- k {x}
```

- Data values

```
t2 <- Cons x xs
```

Side-Effects

- Closures

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t1 <- k {x}
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t2 <- Cons x xs
```

Side-Effects

- Closures

  `t1 <- k {x}`

- Data values

  `t2 <- Cons x xs`

- Primitives

  `t3 <- add*(x, y)`

Sufficiency

- Higher-Order Functions
- Primitives
- Data Values

Related Work

- MLj: Benton, Kennedy, & Russell[1]
- Continuation-Passing Style[2]
- Administrative-Normal Form: Flanagan, Sabry, Duba, and Felleisen[3]

[1] "Compiling Standard ML to Java Bytecodes" (1998).
[2] See Appel's "Compiling with Continuations" (1992).
[3] "The Essence of Compiling with Continuations" (1993).

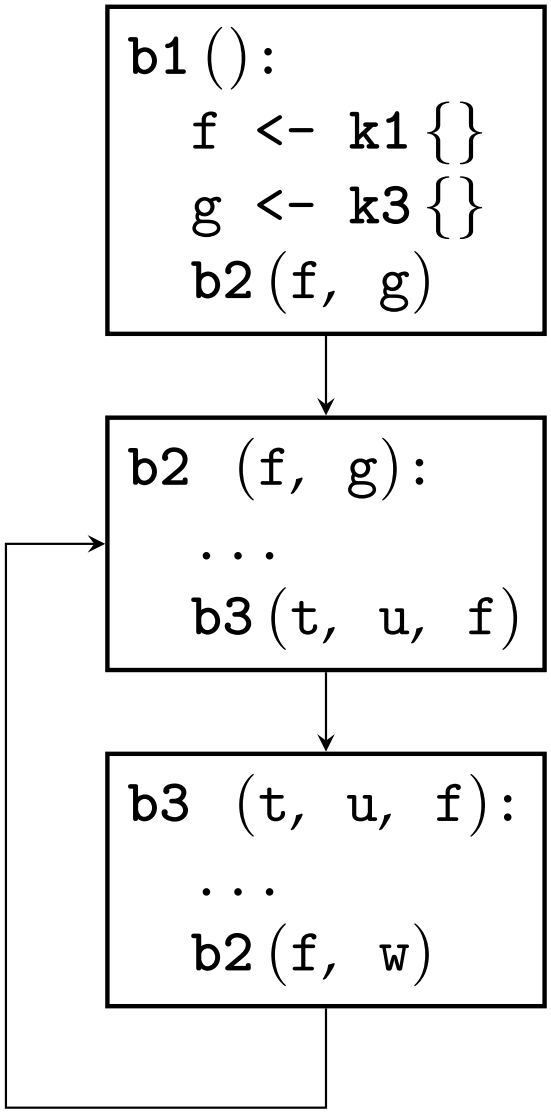
Dataflow Analysis

- Due to Kildall's "A Unified Approach to Global Program Optimization" (1973)
- Widely applied to imperative programming languages

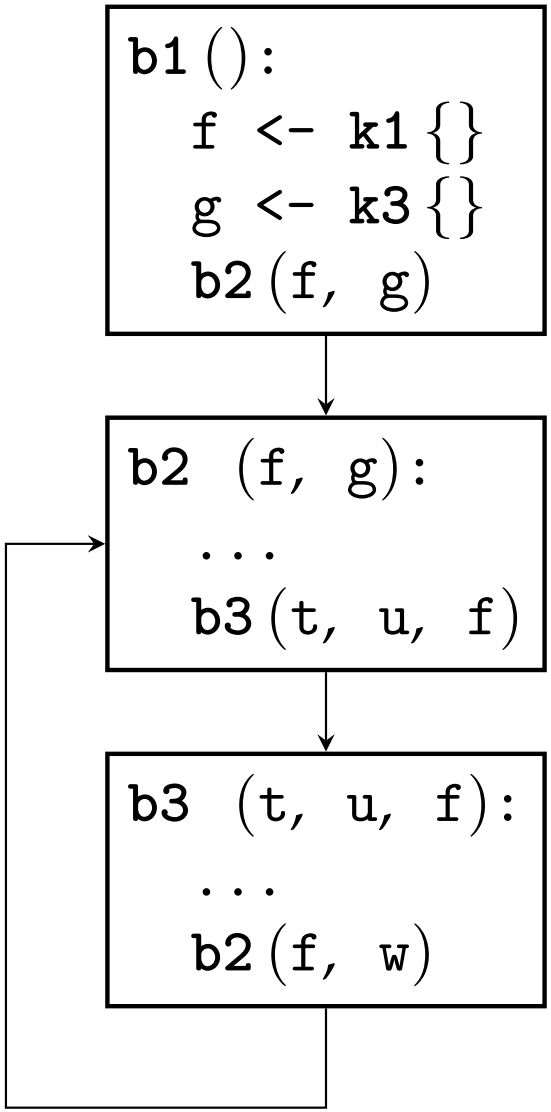
Typical Dataflow Optimizations

- Dead-code Elimination
- Constant Folding
- Lazy Code Motion
- For more, see Muchnick's "Advanced compiler design and implementation" (1997)

Fundamentals: CFGs & Basic Blocks



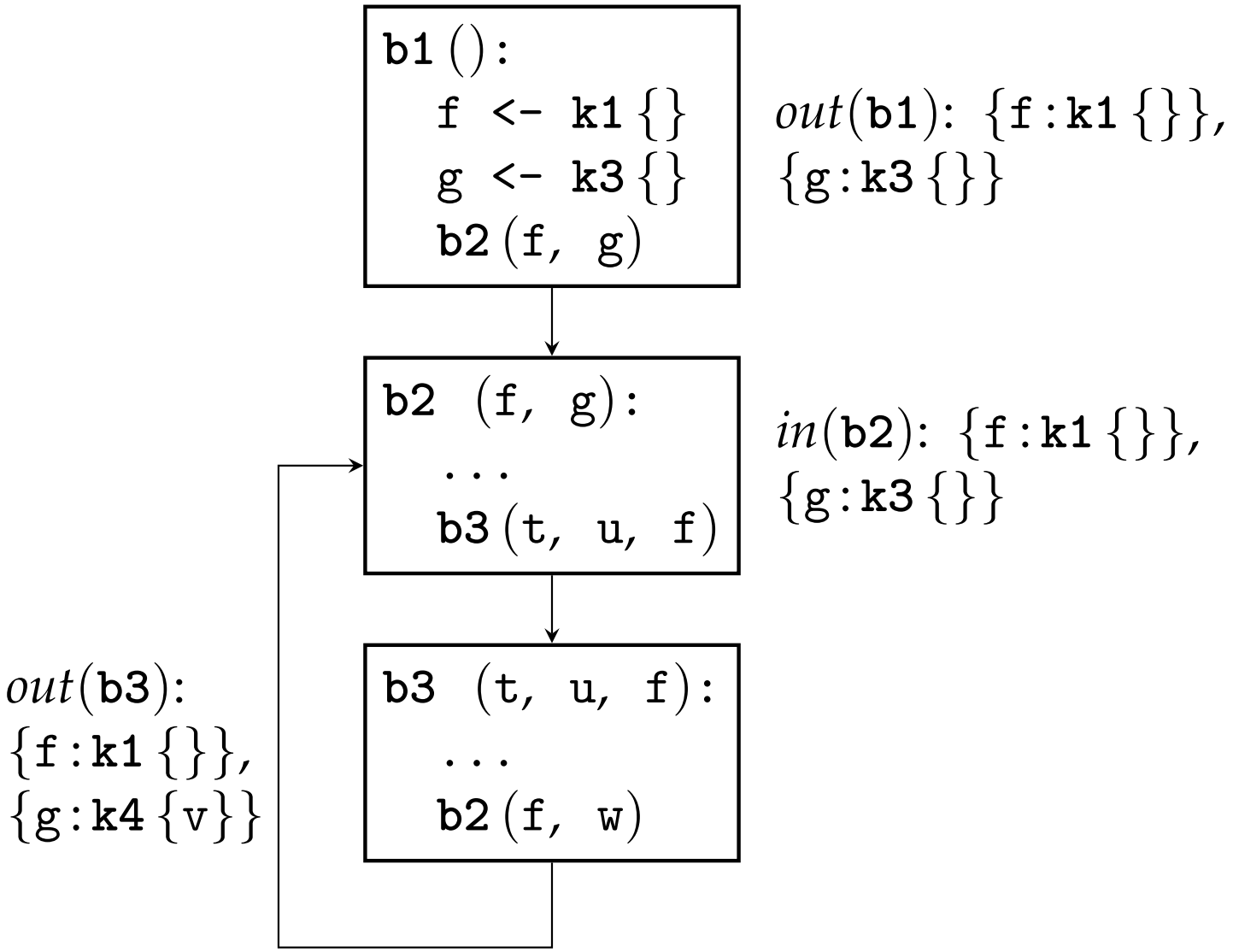
Facts



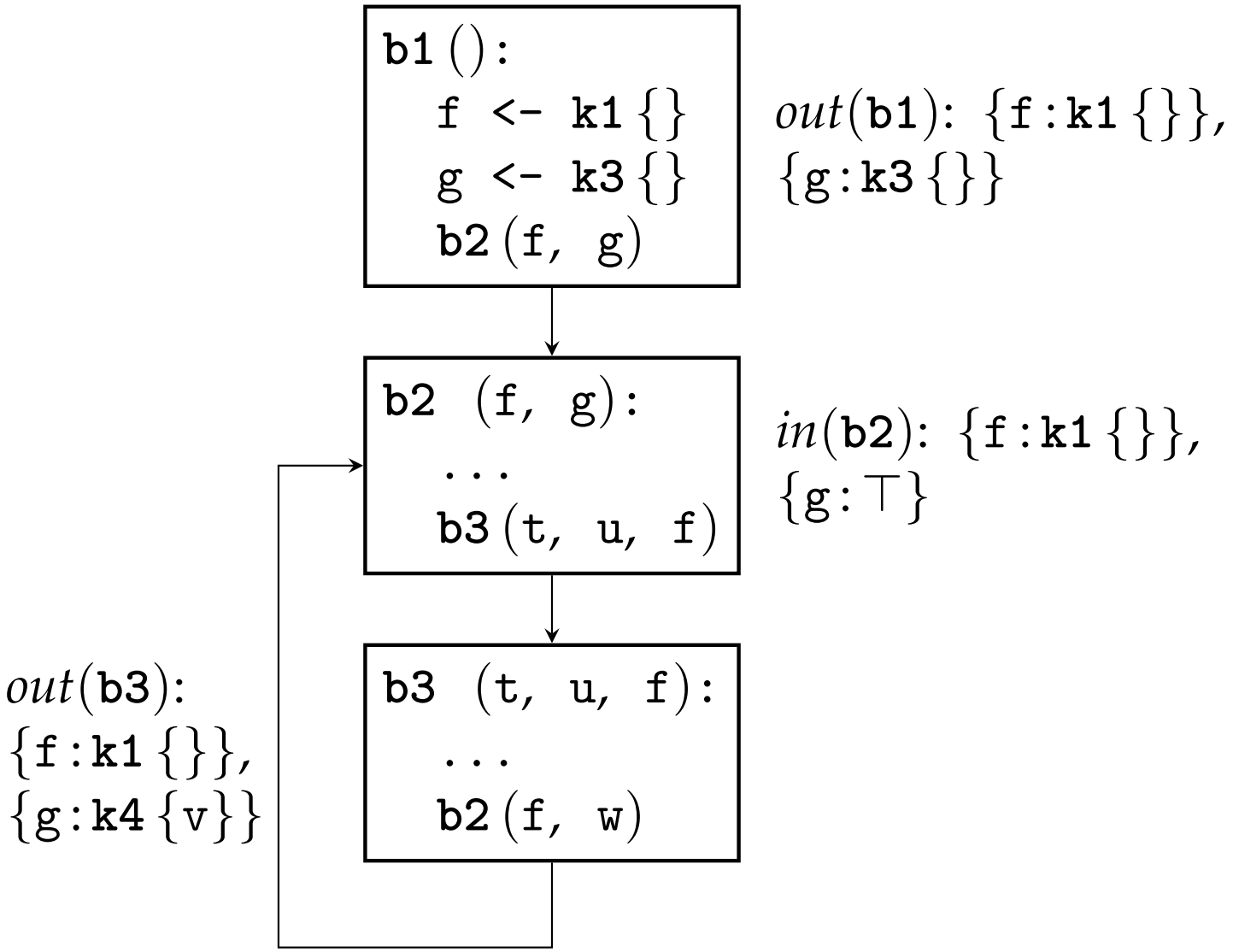
$out(\texttt{b1})$: $\{\texttt{f}:\texttt{k1}\,\{\}\}$, $\{\texttt{g}:\texttt{k3}\,\{\}\}$

$in(\texttt{b2})$: $\{\texttt{f}:\texttt{k1}\,\{\}\}$, $\{\texttt{g}:\texttt{k3}\,\{\}\}$

Iteration


b1 ():
f <- k1 {}
g <- k3 {}
b2 (f, g)
out(b1): {f : k1 {}},
{g : k3 {}}
b2 (f, g):
...
b3 (t, u, f)
in(b2): {f : k1 {}},
{g : k3 {}}
b3 (t, u, f):
...
b2 (f, w)
out(b3):
{f : k1 {}},
{g : k4 {v}}

Iteration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{g : ⊤}
b3 (t, u, f):
...
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out(b3):
{f : k1 {}},
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HOOPL: A Haskell Library for Dataflow Analysis

- See "Hoopl: A Modular, Reusable Library for Dataflow Analysis and Transformation" by Ramsey, Dias, and Peyton Jones (2010)
- Used in the Glasgow Haskell Compiler
- Based on "Composing Dataflow Analyses and Transformations" by Lerner, Grove, and Chambers (2002)

Uncurrying

- Partial Application
- Uncurrying *map*

Partial Application

$$
\begin{aligned}
&map\ f\ xs = \textbf{case}\ xs\ \textbf{of} \\
&\quad Cons\ x\ xs' \rightarrow Cons\ (f\ x)\ (map\ f\ xs') \\
&\quad Nil \rightarrow Nil \\
&toList\ x = Cons\ x\ Nil
\end{aligned}
$$

Partial Application

$$
\begin{aligned}
&mkLists = map\ toList \\
&map\ f\ xs = \textbf{case}\ xs\ \textbf{of} \\
&\quad Cons\ x\ xs' \rightarrow Cons\ (f\ x)\ (map\ f\ xs') \\
&\quad Nil \rightarrow Nil \\
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\end{aligned}
$$

Partial Application

$$
\begin{aligned}
&\mathit{mkLists} = \mathit{map}\ \mathit{toList} \\
&\mathit{main1}\ \mathit{ns} = \mathit{map}\ \mathit{toList}\ \mathit{ns} \\
&\mathit{main2}\ \mathit{ns} = \mathit{mkLists}\ \mathit{ns} \\
&\mathit{map}\ f\ \mathit{xs} = \textbf{case}\ \mathit{xs}\ \textbf{of} \\
&\quad \mathit{Cons}\ x\ \mathit{xs}' \rightarrow \mathit{Cons}\ (f\ x)\ (\mathit{map}\ f\ \mathit{xs}') \\
&\quad \mathit{Nil} \rightarrow \mathit{Nil} \\
&\mathit{toList}\ x = \mathit{Cons}\ x\ \mathit{Nil}
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Uncurrying *map*

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\begin{aligned}
&\mathit{main}\ \mathit{ns} = \mathit{map}\ \mathit{toList}\ \mathit{ns} \\
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$$

```
1  main(ns):
2    v227 <- k203{}
3    v228 <- k219{}
4    v229 <- v227 @ v228
5    v229 @ ns
```

Uncurrying *map*

`main (ns):`

$$\underline{main\ ns} = map\ toList\ ns$$
$$map\ f\ xs = \textbf{case}\ xs\ \textbf{of}$$
$$\quad Cons\ x\ xs' \rightarrow$$
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&\underline{\mathit{toList}\ x} = \mathit{Cons}\ x\ \mathit{Nil}
\end{aligned}
$$

```
1  toList(x):
2    v221 <- Cons1{}
3    v222 <- v221 @ x
4    v223 <- Nil
5    v222 @ v223
```

Uncurrying *map*

`toList (x):`

$$main\ ns = map\ toList\ ns$$
$$map\ f\ xs = \textbf{case}\ xs\ \textbf{of}$$
$$\quad Cons\ x\ xs' \rightarrow$$
$$\quad\quad Cons\ (f\ x)\ (map\ f\ xs')$$
$$\quad Nil \rightarrow Nil$$
$$\underline{toList\ x} = Cons\ x\ Nil$$

```
1  toList (x):
2    v221 <- Cons1 {}
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5    v222 @ v223
```

Uncurrying *map*

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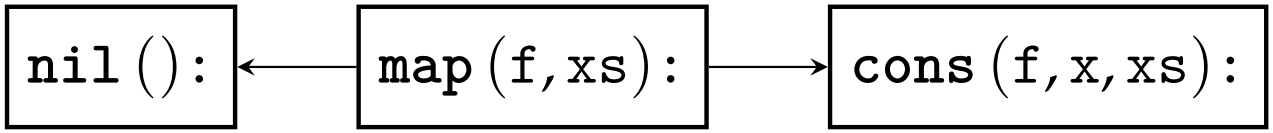
\end{aligned}
$$

```
1  map(f,xs):
2    case xs of
3      Nil -> nil()
4      Cons x xs ->
5        cons(f, x, xs)
```

Uncurrying *map*



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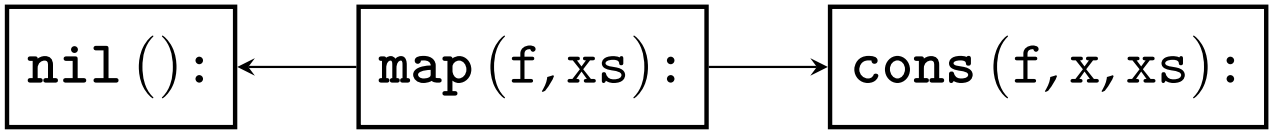

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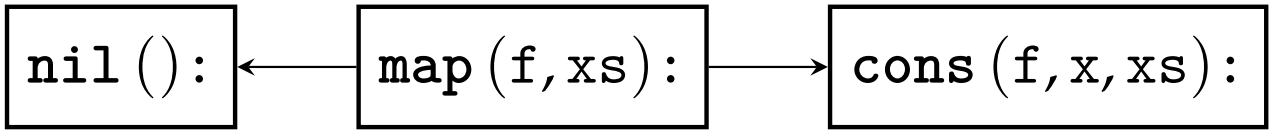
\end{aligned}
$$

```
1  cons (f, x, xs):
2    v209 <- Cons1 {}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203 {}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
```

Uncurrying *map*



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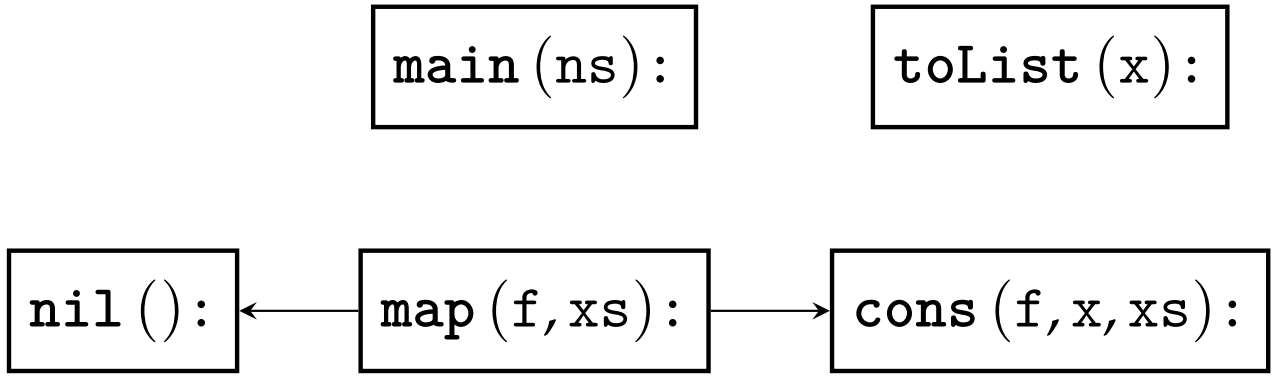
\end{aligned}
$$

```
1  nil(): Nil
```

Uncurrying *map*



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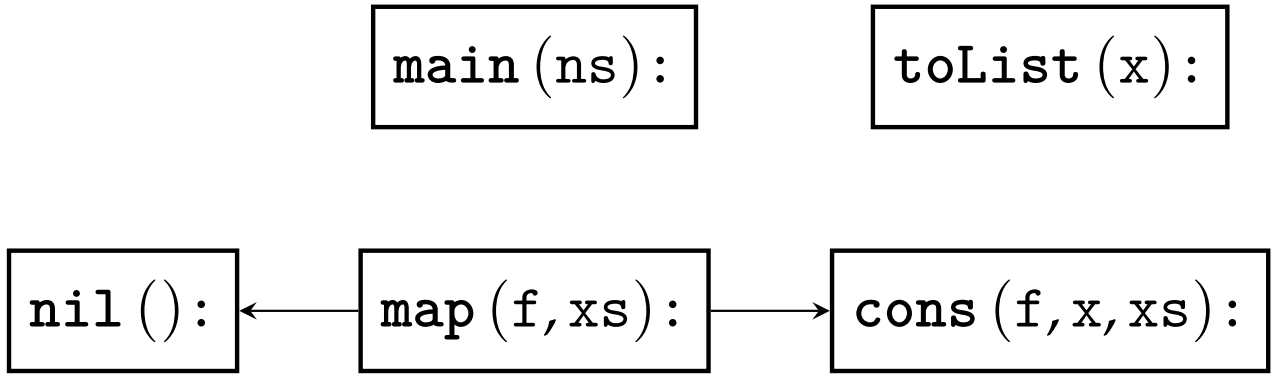


```
1  main (ns):
2    v227 <- k203 {}
3    v228 <- k219 {}
4    v229 <- v227 @ v228
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6  k203 {} f: k204 {f}
7  k204 {f} xs: map (f, xs)
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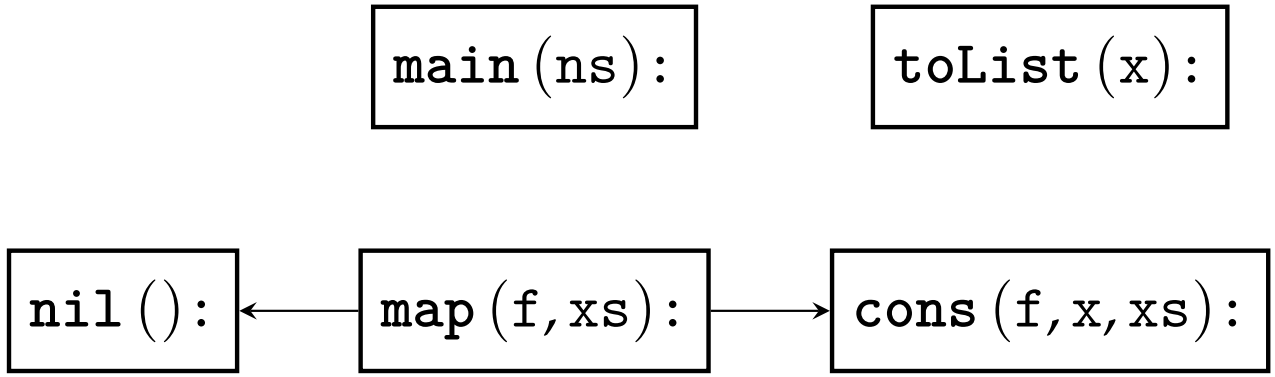
Uncurrying *map*

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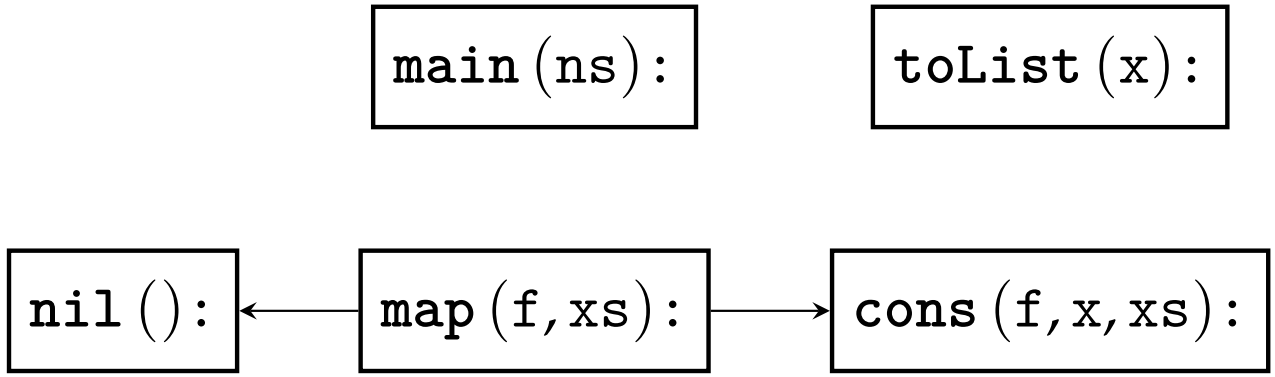


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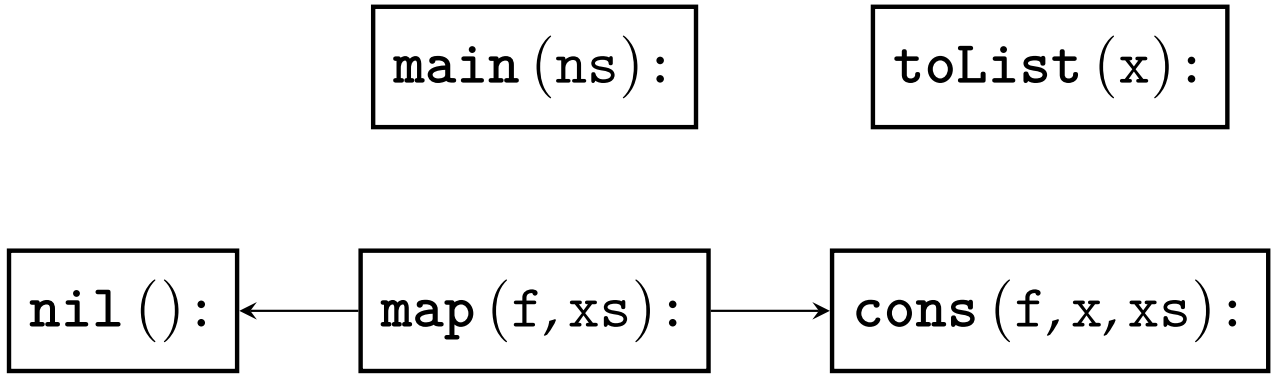


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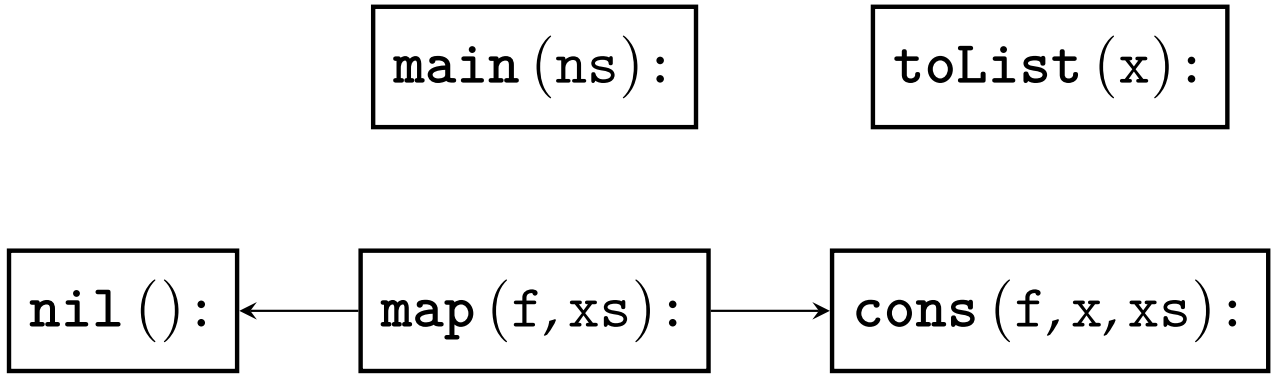


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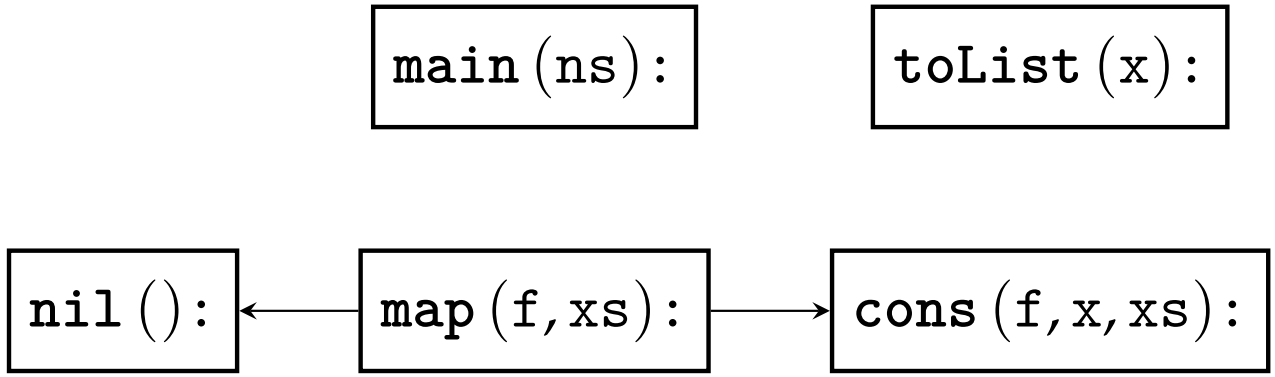


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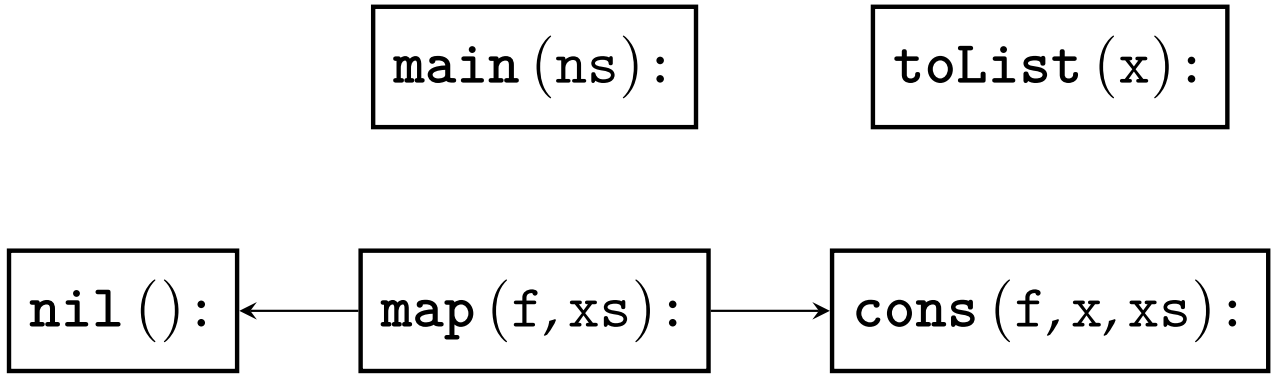


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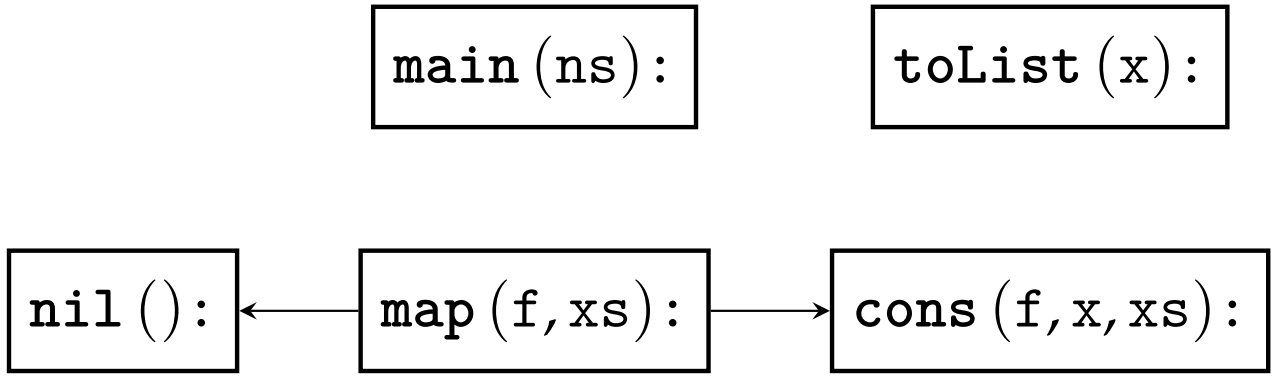


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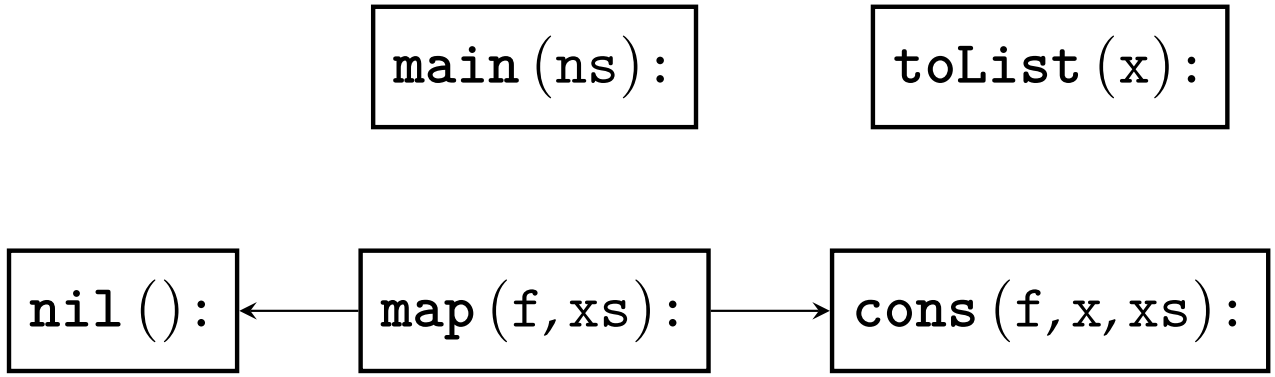


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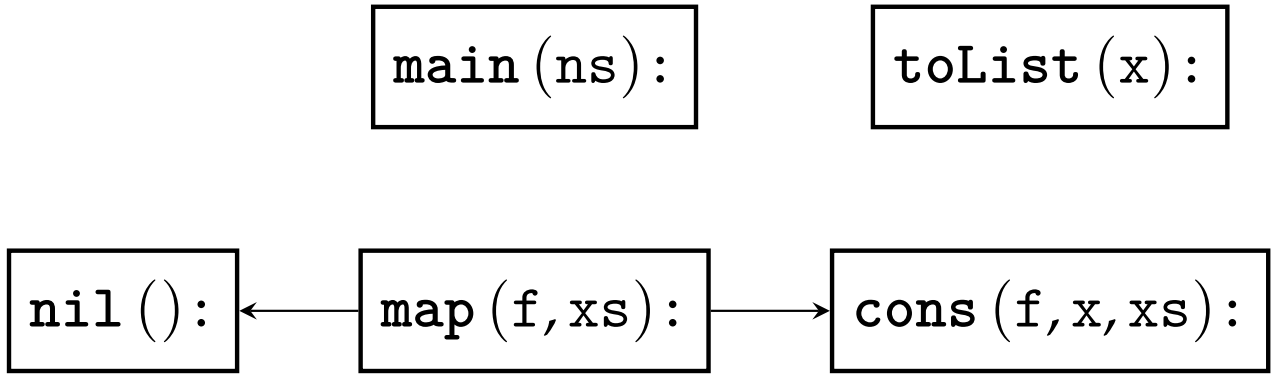


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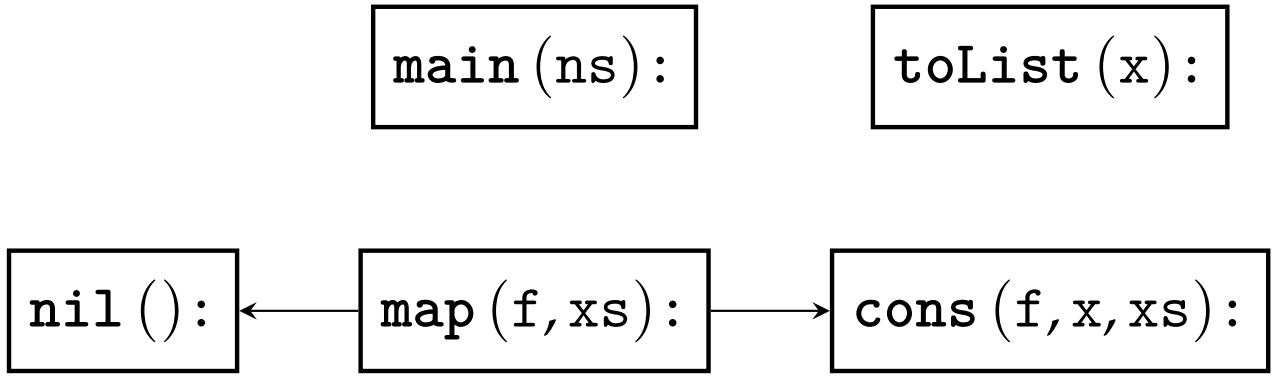


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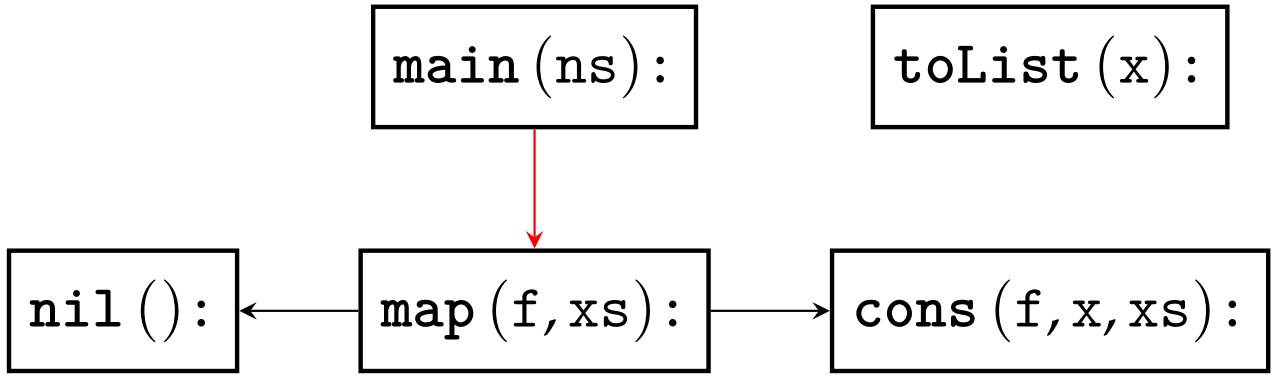


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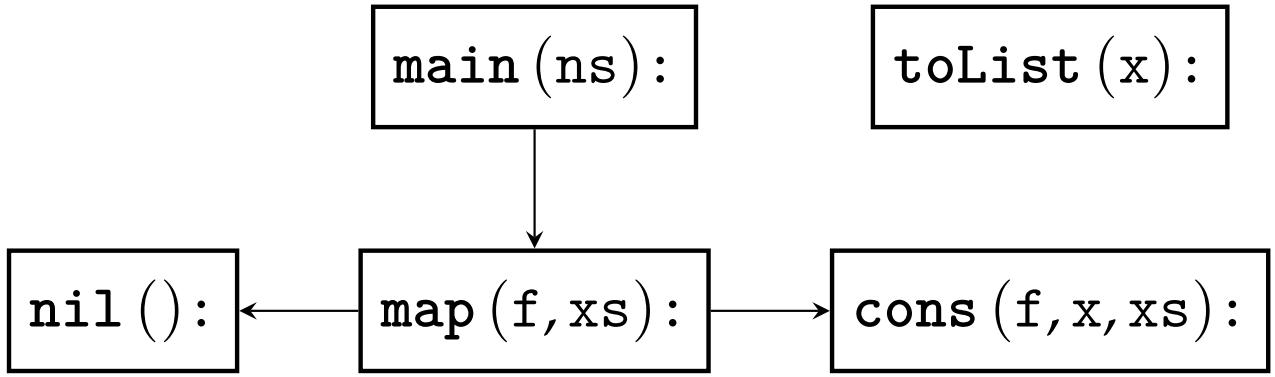
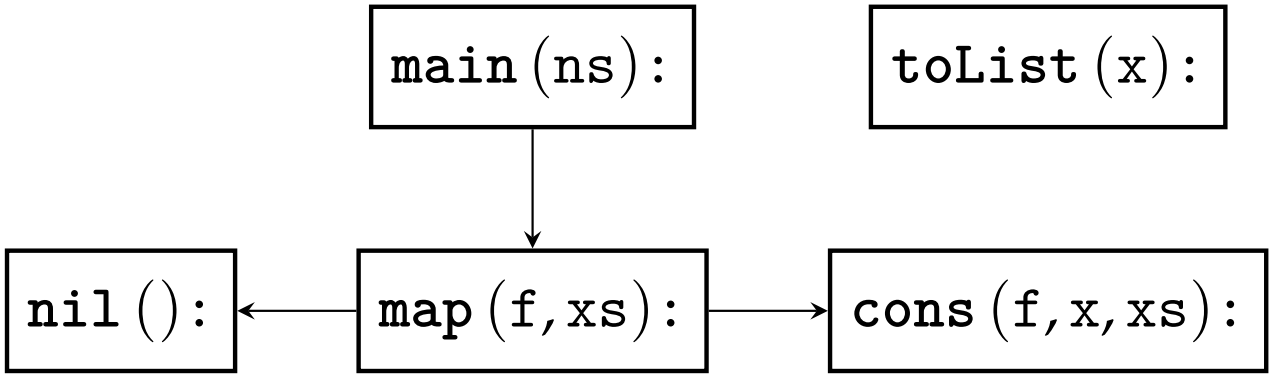


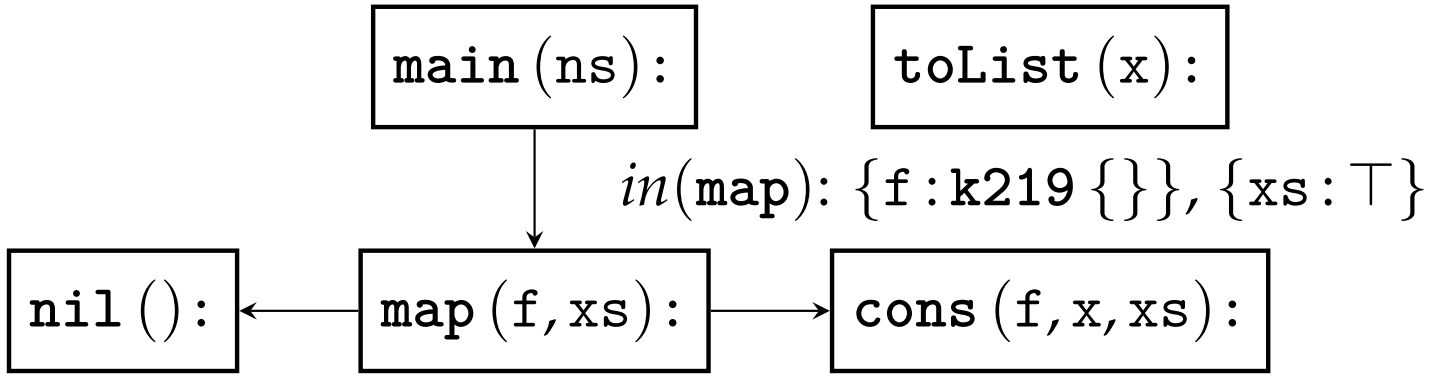

```
1  main (ns): ⟵{ns: ⊤}
2     k203 {}
3     v228 <- k219 {}⟵{v228: k219 {}}
4     k204 {v228}
5     map (v228, ns)
6  k203 {} f: k204 {f}
7  k204 {f} xs: map (f, xs)
8  k219 {} x: toList (x)
```

Uncurrying *map*



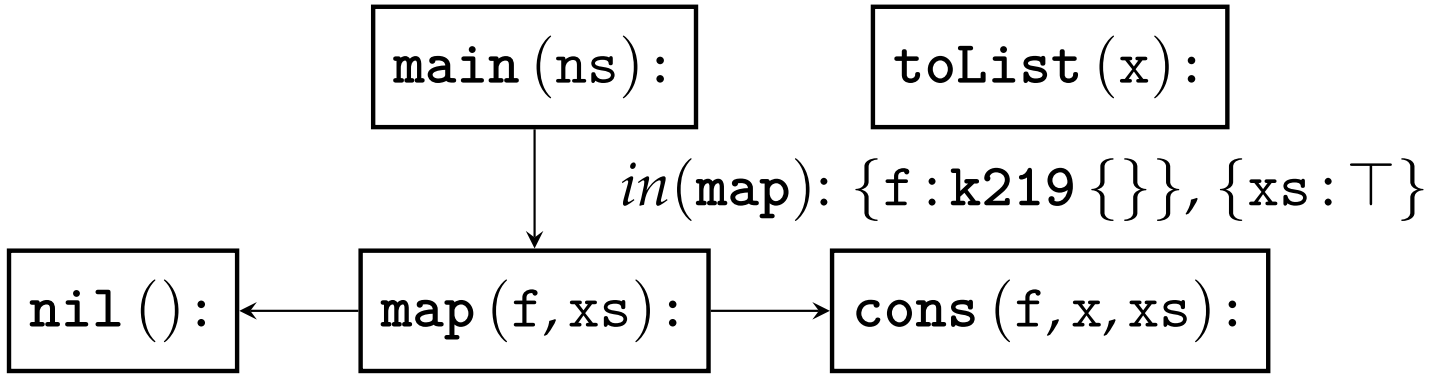

```
1  main (ns): ⟵{ns : ⊤}
2    v228 <- k219 {}⟵{v228 : k219 {}}
3    map (v228, ns)
4  k203 {} f: k204 {f}
5  k204 {f} xs: map (f, xs)
6  k219 {} x: toList (x)
```

Uncurrying *map*



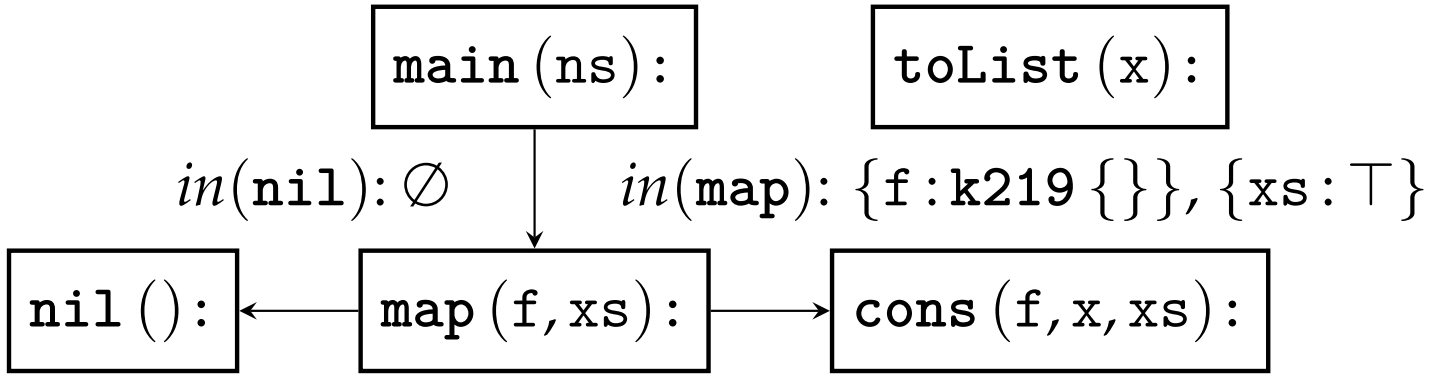

```
1  main (ns):
2    v228 <- k219 {}
3    map (v228, ns)
4  k203 {} f: k204 {f}
5  k204 {f} xs: map (f, xs)
6  k219 {} x: toList (x)
```

Uncurrying *map*



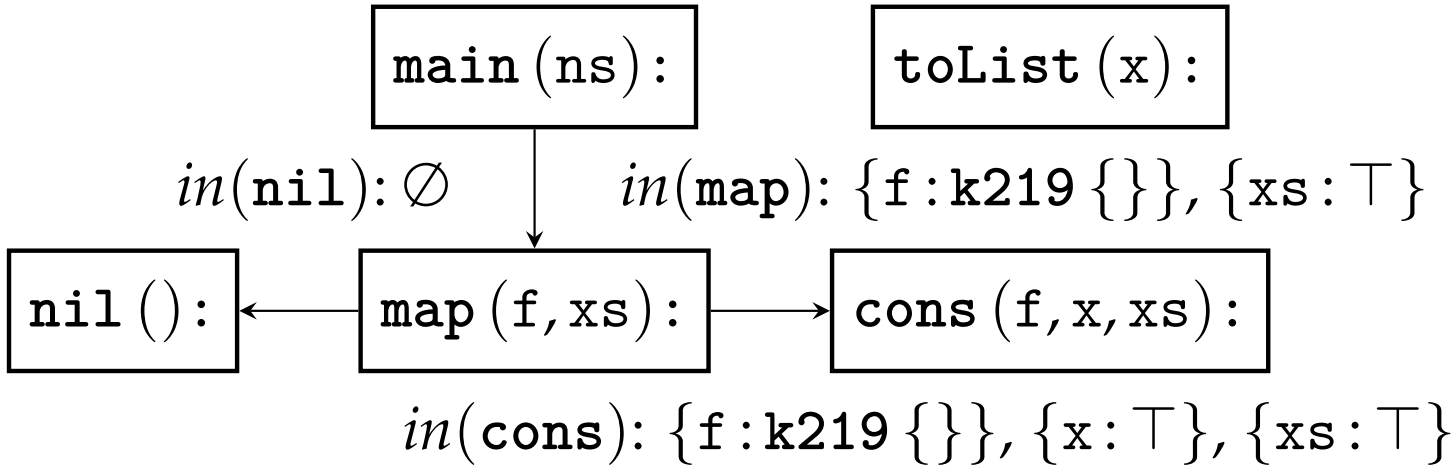

```
1  map (f,xs):                    {f : k219 {}}, {xs : ⊤}
2    case xs of
3      Nil -> nil ()
4      Cons x xs -> cons (f, x, xs)
                    {x : ⊤}, {xs : ⊤}
```

Uncurrying *map*




```
1  map (f,xs):                    {f : k219 {}}, {xs : ⊤}
2    case xs of
3      Nil -> nil ()
4      Cons x xs -> cons (f, x, xs)
                    {x : ⊤}, {xs : ⊤}
```

Uncurrying *map*




```
1  cons (f, x, xs):
2    v209 <- Cons1 {}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203 {}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
9  k219{} x: toList(x)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- k219{} @ x
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
9  k219{} x: toList(x)
```

Uncurrying *map*

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1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x)
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
9  k219{} x: toList(x)
```

Uncurrying *map*

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1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
9  Cons1{} a2: Cons2{a2}
10 Cons2{a2} a1: Cons a2 a1
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons1{} @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
9  Cons1{}a2: Cons2{a2}
10 Cons2{a2}a1: Cons a2 a1
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210}
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
9  Cons1{}a2: Cons2{a2}
10 Cons2{a2}a1: Cons a2 a1
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- v212 @ f ⟵{v213:⊤}
7    v214 <- v213 @ xs ⟵{v214:⊤}
8    v211 @ v214
9  k203{}f: k204{f}
10 k204{f}xs: map(f,xs)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k203{} @ f ⟵{v213:⊤}
7    v214 <- v213 @ xs ⟵{v214:⊤}
8    v211 @ v214
9  k203{}f: k204{f}
10 k204{f}xs: map(f,xs)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- v213 @ xs ⟵{v214:⊤}
8    v211 @ v214
9  k203{}f: k204{f}
10 k204{f}xs: map(f,xs)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- v213 @ xs ⟵{v214:⊤}
8    v211 @ v214
9  k203{}f: k204{f}
10 k204{f}xs: map(f,xs)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- k204{f} @ xs ⟵{v214:⊤}
8    v211 @ v214
9  k203{}f: k204{f}
10 k204{f}xs: map(f,xs)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- map(f,xs) ⟵{v214:⊤}
8    v211 @ v214
9  k203{}f: k204{f}
10 k204{f}xs: map(f,xs)
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- map(f,xs) ⟵{v214:⊤}
8    v211 @ v214
9  Cons1{}a2: Cons2{a2}
10 Cons2{a2}a1: Cons a2 a1
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- map(f,xs) ⟵{v214:⊤}
8    Cons2{v210} @ v214
9  Cons1{}a2: Cons2{a2}
10 Cons2{a2}a1: Cons a2 a1
```

Uncurrying *map*

```
1  cons(f, x, xs): ⟵{f:k219{}}, {x:⊤}, {xs:⊤}
2    v209 <- Cons1{} ⟵{v209:Cons1{}}
3    v210 <- toList(x) ⟵{v210:⊤}
4    v211 <- Cons2{v210} ⟵{v211:Cons2{v210}}
5    v212 <- k203{} ⟵{v212:k203{}}
6    v213 <- k204{f} ⟵{v213:k204{f}}
7    v214 <- map(f,xs) ⟵{v214:⊤}
8    Cons v210 v214
9  Cons1{}a2: Cons2{a2}
10 Cons2{a2}a1: Cons a2 a1
```

Uncurrying *map*

```
1  cons(f, x, xs):
2    v209 <- Cons1{}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203{}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
```

Uncurrying *map*

```
1  cons(f, x, xs):
2    v209 <- Cons1{}
3    v210 <- toList(x)
4    v211 <- Cons2{v210}
5    v212 <- k203{}
6    v213 <- k204{f}
7    v214 <- map(f,xs)
8    Cons v210 v214
```

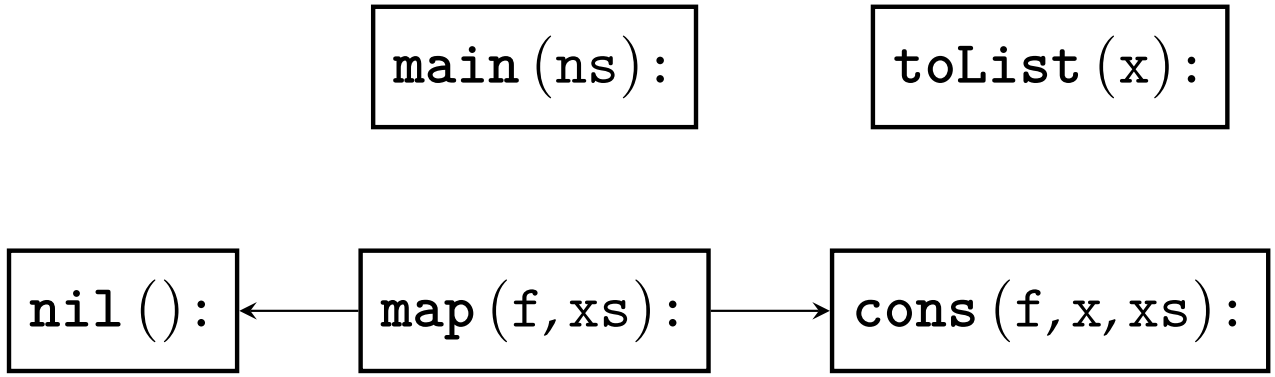
Uncurrying *map*

```
1  cons(f, x, xs):
2    ~~v209 <- Cons1~~
3    v210 <- toList(x)
4    ~~v211 <- Cons2 v210~~
5    ~~v212 <- k203~~
6    ~~v213 <- k204 f~~
7    v214 <- map(f,xs)
8    Cons v210 v214
```

Uncurrying *map*

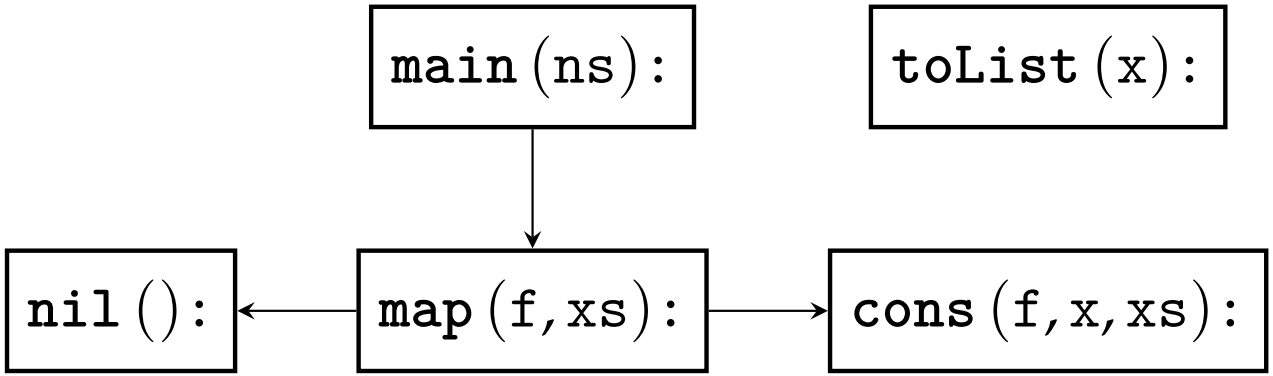
```
1  cons(f, x, xs):
2    v210 <- toList(x)
3    v214 <- map(f,xs)
4    Cons v210 v214
```

Uncurrying *map*



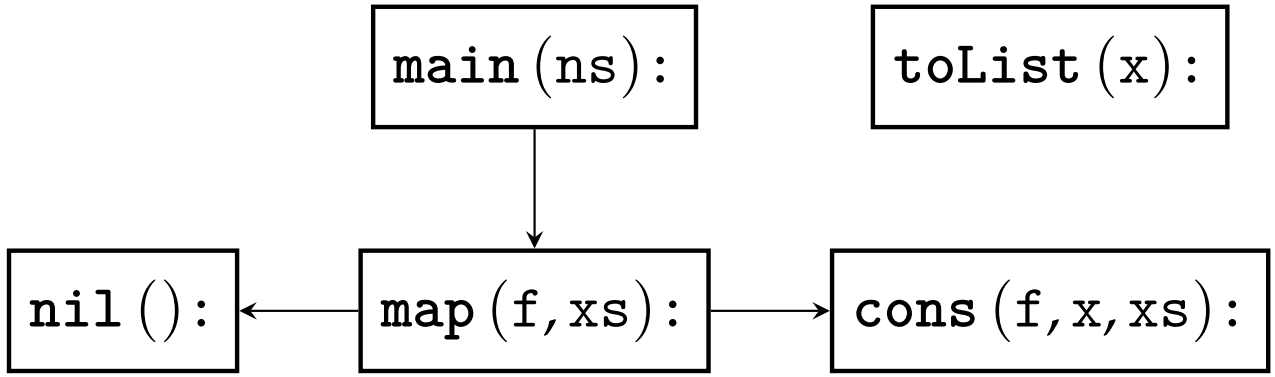

```
1  main (ns):
2     v227 <- k203 {}
3     v228 <- k219 {}
4     v229 <- v227 @ v228
5     v229 @ ns
```

Uncurrying *map*



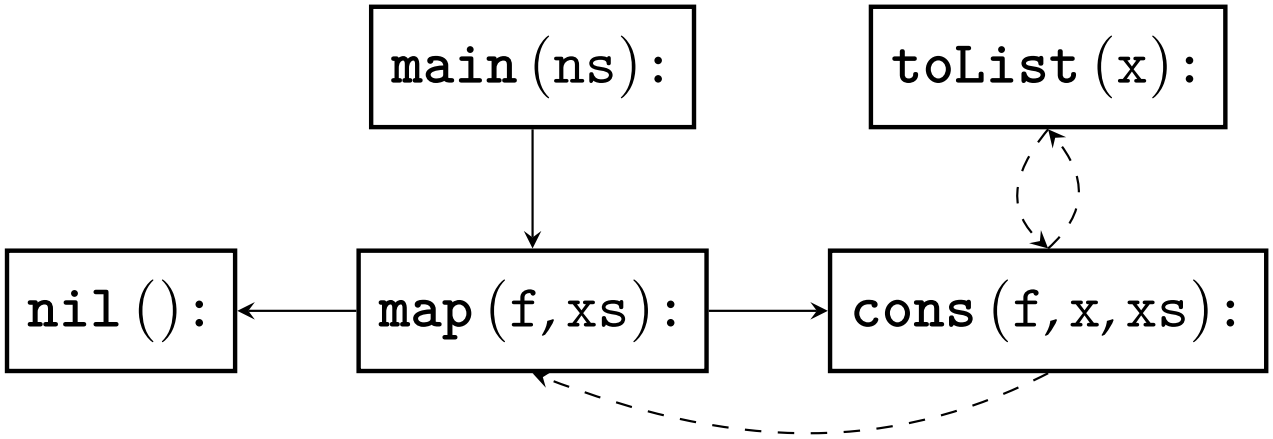

```
1  main (ns):
2      k217 = k203 {}
3      v228 <- k219 {}
4      k229 = k217 {} v228
5      map (v228, ns)
```

Uncurrying *map*



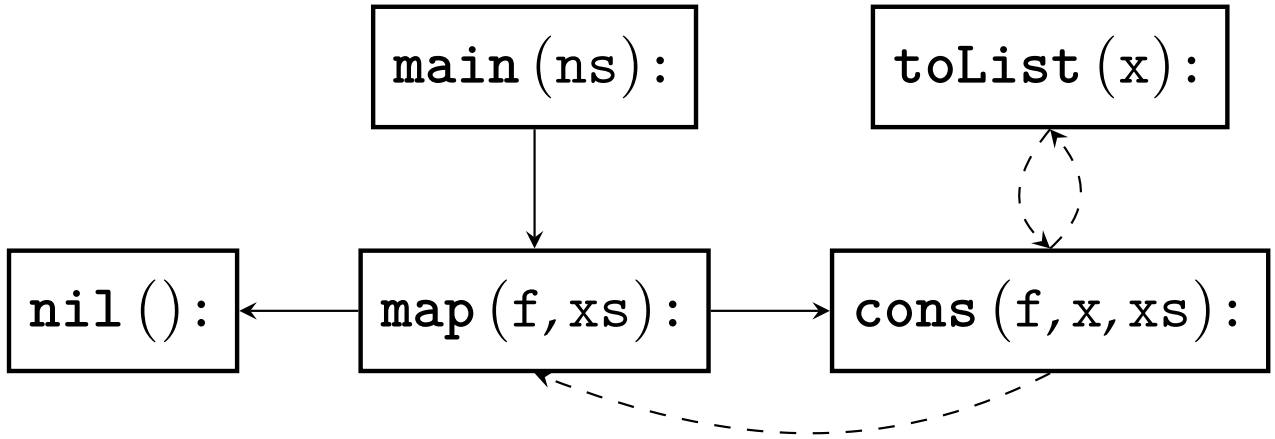

```
1  cons (f, x, xs):
2    v209 <- Cons1 {}
3    v210 <- f @ x
4    v211 <- v209 @ v210
5    v212 <- k203 {}
6    v213 <- v212 @ f
7    v214 <- v213 @ xs
8    v211 @ v214
```

Uncurrying *map*




```
1  cons (f, x, xs):
2    ~~v209 <- Cons x {}~~
3    v210 <- toList (x)
4    ~~v211 <- v209 @ v210~~
5    ~~v212 <- &v203 {}~~
6    ~~v213 <- v212 @ f~~
7    v214 <- map (f,xs)
8    Cons v210 v214
```

Uncurrying *map*



- One closure and two applications eliminated from `main`.
- Link from `main` to `map`.
- Two closures and four function applications from `cons`.
- Link from `cons` to `map` and `toList`.

Related Work

- Appel: Uncurrying by pattern matching; "Compiling with Continuations" (1992)
- Tarditi: Uncurrying in four passes; "Design and Implementation of Code Optimizations for a Type-Directed Compiler for Standard ML" (1996)
- Tolmach & Oliva: Automatic uncurrying; "From ML to Ada: Strongly-typed Language Interoperability via Source Translation" (1998)

Conclusion

- Monadic Optimizations
- Contributions

Optimizing using the Monad Laws

- *Left-Unit*

$$\mathbf{do}\,\{x \leftarrow return\ y; m\} \equiv\ \mathbf{do}\,\{[y \mapsto x]\ m\}$$

- *Right-Unit*

$$\mathbf{do}\,\{x \leftarrow m; return\ x\} \equiv\ \mathbf{do}\,\{m\}$$

- *Associativity*

$$\mathbf{do}\,\{x \leftarrow \mathbf{do}\,\{y \leftarrow m; n\}; o\} \equiv\ \mathbf{do}\,\{y \leftarrow m; x \leftarrow n; o\}$$

- From Wadler's "Monads for Functional Programming" (1995)

Dataflow Analysis & MIL

- High-level functional programming; low-level details.
- Structured for dataflow analysis.
- Implemented other optimizations; for example, dead-code elimination.

Uncurrying

- Implemented using the dataflow algorithm
- Able to uncurry across blocks and loops (with some caveats)
- Complete implementation described

HOOPL

- Thorough description of the library
- Simple, but complete, example implementation given

Questions?

Source code and paper available at `http://mil.codeslower.com`, or email me at `jgbailey@codeslower.com`.

Related Work: Appel

$$f\ x = g\ x$$
$$g\ x\ y = x + y$$
$$main\ a\ b = f\ a\ b$$

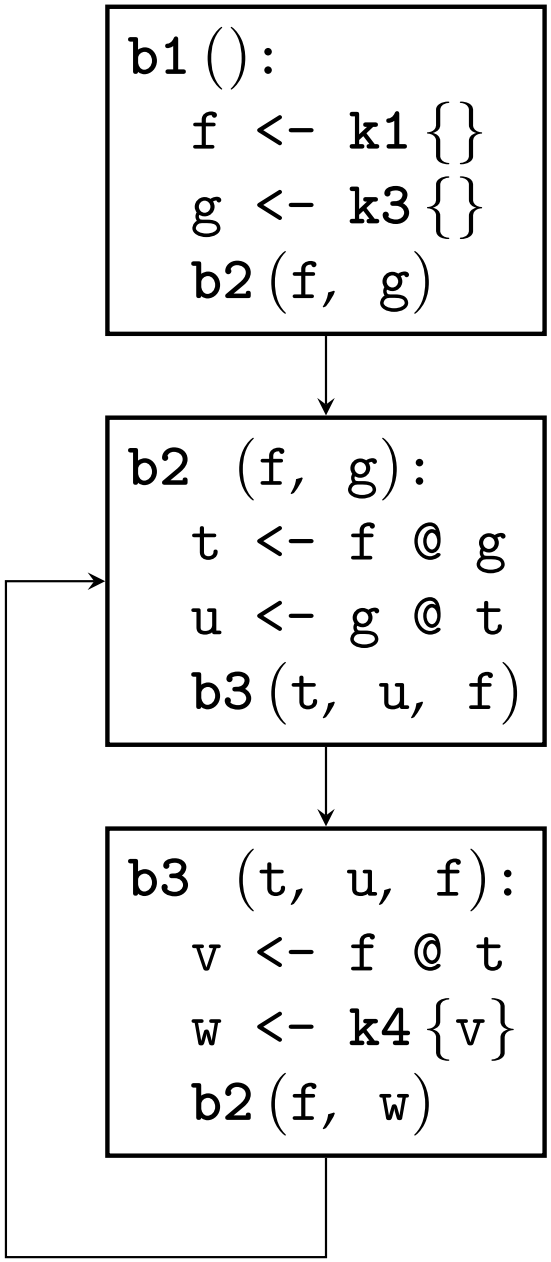
```
1  main(a, b): b208(a, b)
2  b208(x, y):
3    v210 <- plusclo1{x}
4    v210 @ y
5  plusclo1{a2}a1: plus*(a2, a1)
6  f(): k212{}
7  k212{}x: k207{x}
8  g(): k206{}
9  k206{}x: k207{x}
10 k207{x}y: b208(x, y)
```

Related Work: Tarditi

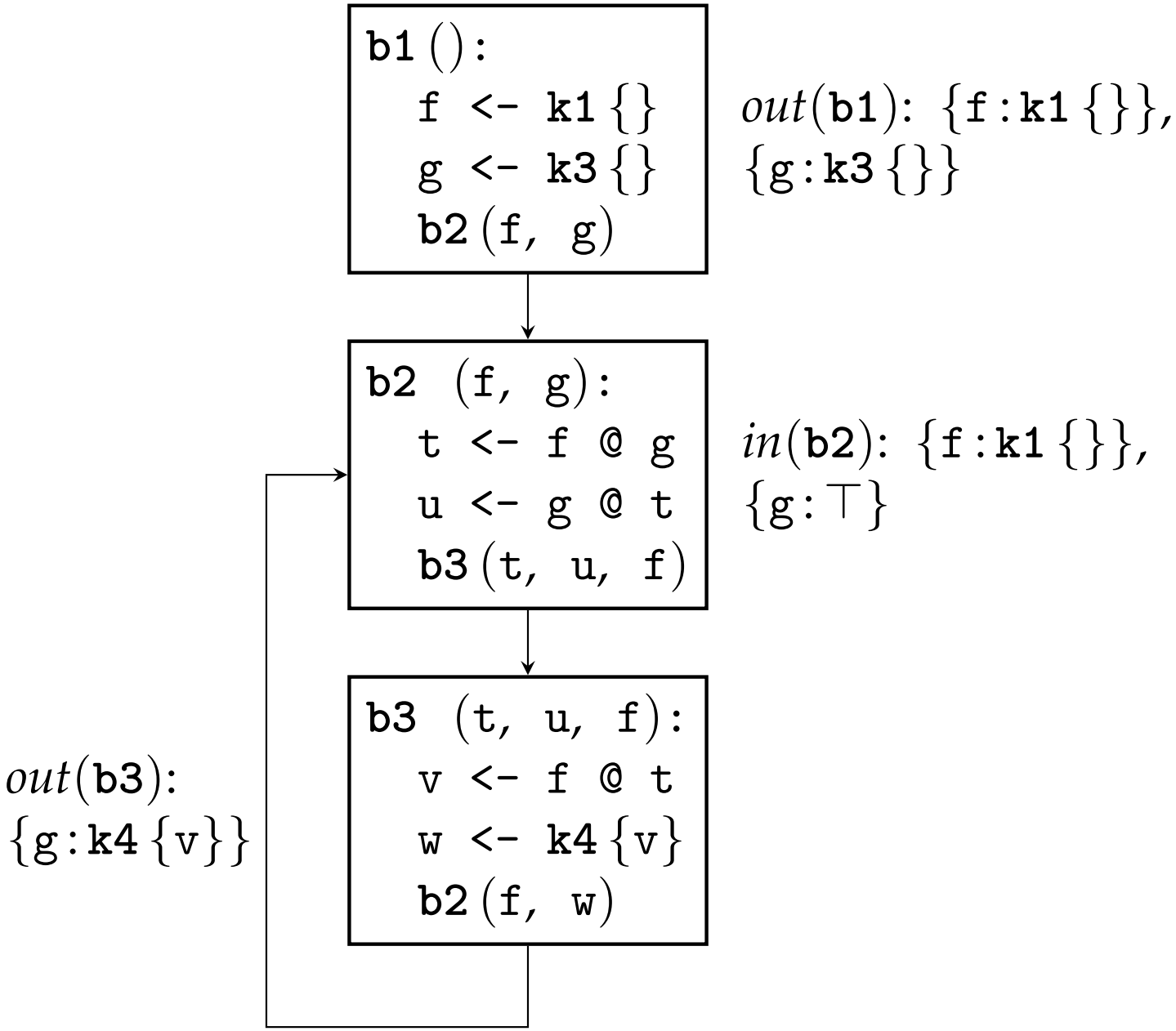
$$g\ x\ y\ z = x + y + z$$
$$h\ x\ y = g\ x\ y$$
$$main\ s\ t\ u = h\ s\ t\ u$$

```
1  main(s, t, u): b216(s, t, u)
2  b216(x, y, z):
3    v219 <- plusclo1{x}
4    v220 <- v219 @ y
5    v221 <- plusclo1{v220}
6    v221 @ z
7  plusclo1{a2} a1: plus*(a2, a1)
8  g{} : k213{}
9  k213{} x: k214{x}
10 k214{x} y: k215{x, y}
11 h{} : k207{}
12 k207{} x: k208{x}
13 k208{x} y: k215{x, y}
14 k215{x, y} z: b216(x, y, z)
```

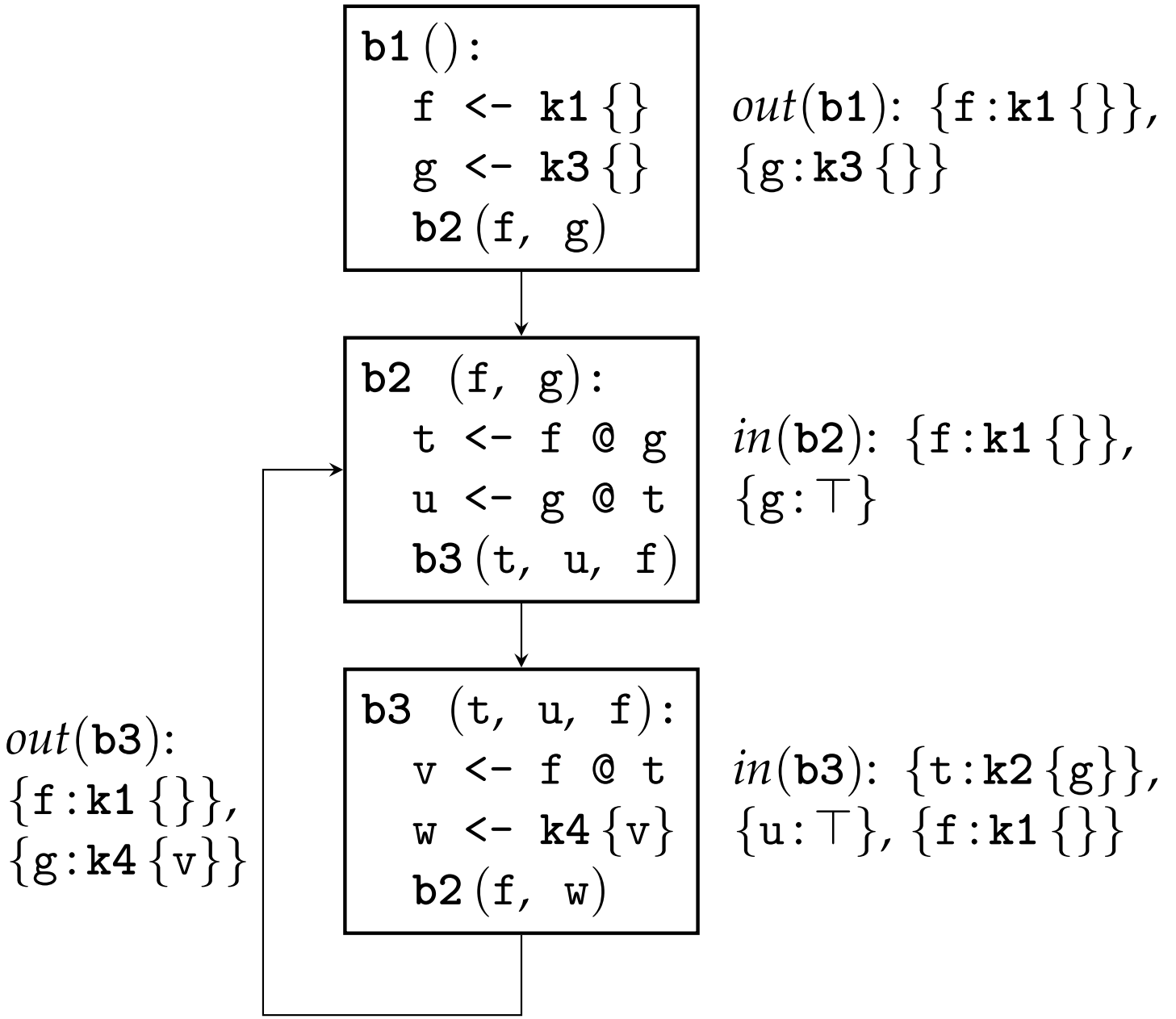
Original Program



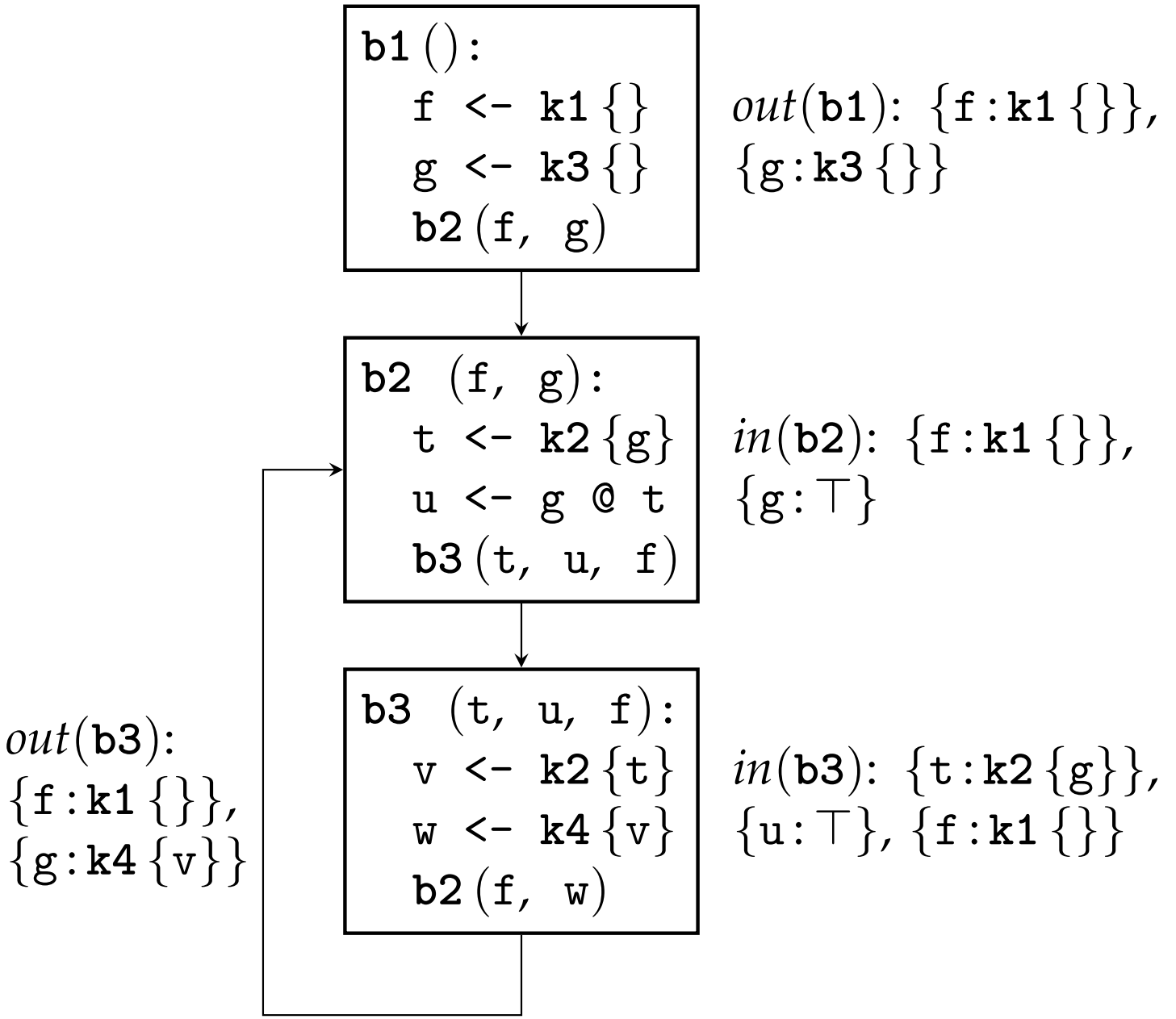
Initial Facts



Facts After Iteration



Rewrite


b1 ():
f <- k1 {}
g <- k3 {}
b2 (f, g)
out(b1): {f : k1 {}},
{g : k3 {}}
b2 (f, g):
t <- k2 {g}
u <- g @ t
b3 (t, u, f)
in(b2): {f : k1 {}},
{g : ⊤}
b3 (t, u, f):
v <- k2 {t}
w <- k4 {v}
b2 (f, w)
in(b3): {t : k2 {g}},
{u : ⊤}, {f : k1 {}}
out(b3):
{f : k1 {}},
{g : k4 {v}}

Future Work

- Eliminating Thunks
- "Push Through Cases"